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# The Heterotic Life of the D-particle.

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## Abstract

We study the dynamics of D-particles (D0-branes) in type I' string theory and of the corresponding states in the dual heterotic description. We account for the presence of the two 8-orientifolds (8 dimensional orientifold planes) and sixteen D8-branes by deriving the appropriate quantum mechanical system. We recover the familiar condition of eight D8-branes for each 8-orientifold. We investigate bound states and compute the phase shifts for the scattering of such states and find that they agree with the expectations from the supergravity action. In the type I' regime we study the motion transverse to the 8-orientifold and find an interesting cancellation effect.

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# 1 Introduction

String solitons carrying RR charges, known as D-branes, [1, 2] have proven to be of the utmost importance in testing and making use of the various string dualities that have emerged in the last two years [3, 4, 5, 6, 7, 8, 9, 10, 11]<sup>3</sup>. More recently, these D-branes have also provided probes [30, 31, 32, 33, 34, 35, 36] of the geometry of spacetime.

The simplest of all D-branes is obviously the D-particle (D0-brane) in type IIA string theory. From the ten dimensional point of view it corresponds to a charged, extremal black hole, while from the eleven dimensional M-theory point of view it is a Kaluza-Klein state where the quantized charge is related to the quantization of momentum in the compact direction. The D-particle is just an eleven dimensional graviton traveling around a circle.

Clearly such a system deserves a careful study. Perhaps important lessons can be learnt about the elusive M-theory? Such a study has been initiated in [30, 31, 35]. There it was found that not only the mass and charge of the D-particle has an eleven dimensional interpretation but also the dynamics is best understood in these terms. The characteristic length and energy scales are both eleven dimensional. This suggests the fascinating possibility that D-particles might be used to *formulate* M-theory, [37]. A necessary condition for this idea to be applicable is Hull's analysis [38] suggesting that these objects dominate in the strong coupling limit of the type IIA string.

In this paper we continue the study in the case of configurations relevant to the type I' and heterotic theories. To do this we construct the type I' theory as a type IIA orientifold [39, 40, 41, 42], where the last space coordinate is a  $S^1/Z_2$  orbifold. The type I' D-particle is then the T-dual of the type I D-string. We study how such D-particles interact in this environment. By using type I/heterotic duality [6, 8, 11] we can identify these states with the Kaluza-Klein modes of the heterotic string around a circle and investigate some of their properties as well.

The outline and the main results of the paper are as follows. In the next section, we begin with a review of the type IIA D-particle. We point out some of the most interesting results obtained in previous work, in particular,

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<sup>3</sup>An early review of string solitons is [12], some recent reviews of various aspects of duality are [13, 14, 15, 16]. Dirichlet boundary conditions for open strings have been studied, for instance, in [17, 18, 19]. Among the many applications of D-brane techniques, we should also mention black hole entropy [20, 21, 22, 23, 24, 25] and string/D-brane interaction (e.g. gravitational lensing) [26, 27, 28, 29].

the importance of the eleven dimensional Planck scale. In section three, we derive the quantum mechanical system that describes the interactions of the type I' D-particles. In section four we discuss the physical interpretation of this model; we identify the flat directions and argue about the existence of non BPS resonances. In section five we present the results in terms of effective potentials. We study the scattering of such states against the 8-orientifold and against each other. The potential is  $O(v^2)$  and  $O(v^4)$  respectively. In section six we try to extrapolate the results to the heterotic string using string dualities. We see that the dynamics is reproduced by the supergravity action. There, we also find that the eleven dimensional Planck scale plays an important role in the type I' theory and speculate that the same might happen in the heterotic limit.

## 2 Review of the type IIA D-particle

In this section we briefly review some of the results of [30, 31, 35] concerning the type IIA D-particle. Consider a collection of  $N$  such D-particles; their low energy, short distance, effective action [43] is given by the quantum mechanical system obtained by the dimensional reduction [44] of a (9+1)-dimensional  $U(N)$  Super Yang-Mills theory all the way down to 0+1 dimension. The  $U(1)$  subgroup factors out and describes the free motion of the center of mass, leaving a non trivial  $SU(N)$  dynamics. The reduced system has 16 supercharges<sup>4</sup>

$$Q_\alpha = \sqrt{\lambda_{IIA}} \gamma_{i\alpha\beta} \psi_\beta^a E_i^a - \frac{1}{2\sqrt{\lambda_{IIA}}} f^{abc} \gamma_{ij\alpha\beta} \psi_\beta^a A_i^b A_j^c, \quad (1)$$

anticommuting to give the hamiltonian

$$H = \frac{\lambda_{IIA}}{2} E_i^{a2} - \frac{1}{2} i f^{abc} A_i^a \psi^b \gamma_i \psi^c + \frac{1}{4\lambda_{IIA}} (f^{abc} A_i^b A_j^c)^2 \quad (2)$$

up to a gauge transformation.

The hamiltonian (2) should have one marginally stable bound state at threshold as required by the relation between type IIA and M-theory [4, 5, 9]. While by now nobody doubts this fact, mostly due to the arguments of Sen

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<sup>4</sup>The notation is as in [30] and will not be repeated;  $E_i^a$  is the canonical conjugate of  $A_i^a$ ,  $\psi^a$  are 16 component spinors satisfying "gamma matrix" type anticommutation relations and  $f^{abc}$  are the structure constants of  $SU(N)$ .

[45, 46], a “direct” proof of this fact is still missing. What *can* be done is to show the existence of resonances whose energy depends on the string coupling as  $E \approx (\lambda_{IIA})^{1/3}$  in string units (i.e.  $\alpha'_{IIA} = 1$ ). As the binding is due to open strings stretching between pairs of D-particles and each string contributes with a potential that grows linearly with the separation, the average size of such resonances is  $r \approx (\lambda_{IIA})^{1/3}$ . Also, as the mass of each constituent is  $1/\lambda_{IIA}$ , the orbital motion is non relativistic at weak coupling, with typical orbital velocity  $v \approx \lambda_{IIA}^{2/3}$ .

This is consistent with the picture emerging from the one loop string calculation of [47] (see also [48, 49, 50, 51]). There, one studies the scattering of two D-branes and sees the growth of the absorptive part  $\text{Im}(\delta)$  of the phase shift as the impact parameter  $r$  becomes shorter than the square root of the velocity<sup>5</sup> (always in string units). This is precisely the same scaling found for the above resonances,  $r \approx \sqrt{v} \approx \lambda_{IIA}^{1/3}$ , confirming the idea that these are the states responsible for the inelasticity.

An even more intriguing outcome of the calculation is the appearance of the 11 dim Planck length  $l_{11} \approx \lambda_{IIA}^{1/3}$  as a dynamical length. A hint that the D-particle “remembers” its 11 dimensional origin can already be seen by expressing the resonance energies in units appropriate to the 11 dimensional theory as in [30], but definitive evidence was presented in [31, 35] by pointing out that there is a range of velocities where the D-particles have non relativistic dynamics and energies below the string mass while having momentum of order  $1/l_{11}$ . At weak coupling,  $l_{11}$  is shorter than the string length, indicating that D-particles (actually, D-branes in general) may be used to probe sub-stringy distances in the spirit of an earlier proposal of Shenker<sup>6</sup>[52].

In [35] the scattering of two D-particles was also studied in the eikonal (i.e. background gauge) approximation, also showing agreement with the calculation of Bachas [47]. The main physical results were that the moduli space for such objects remains flat<sup>7</sup> and that  $l_{11}$  also governs the scattering amplitude via

$$f(k, \theta) \approx e^{-\sqrt{2} \sin(\theta/2) (kl_{11})^{3/2}}. \quad (3)$$

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<sup>5</sup>This region is referred to as “the stadium” in [35].

<sup>6</sup>However, the 11 dim Planck length is still larger than Shenker’s original proposal for a minimal length that was  $l_{min} \approx \lambda_{IIA}$ . This length coincides with the 4 dim Planck length and, amusingly, with the M-theory compactification radius  $R_{11}$  in string units.

<sup>7</sup>I.e., the potential between two type IIA D-particles is  $O(v^4)$  due to the large number of supersymmetries.

We shall see the counterpart of all these statements in the type I' and Heterotic case.

### 3 The type I' D-particle

In this section we work out the lagrangian and the hamiltonian descriptions for the type I' D-particle to be used for dynamical studies in the following sections. We begin with identifying the appropriate degrees of freedom by a straightforward application of the formalism developed in [42]. The type I' theory is an orientifold quotient of type IIA theory obtained by T-dualizing the usual construction of type I string theory as an orientifold of type IIB:

$$I = \frac{IIB}{\{1, \Omega\}} \xrightarrow{T} I' = \frac{IIA}{\{1, \Omega R\}}. \quad (4)$$

In both cases the orientifold group is isomorphic to  $Z_2$ , where the non trivial element is simply world-sheet parity  $\Omega$  in the type IIB case and the composition of world-sheet parity with the reflection  $R$  along, say,  $X^9$  in the type IIA case. The direction  $X^9$  is assumed to be compactified on a circle and it is on this circle that T-duality acts as usual by inverting the radius. The two fixed points of the action of the reflection  $R$  on the circle are identified as the positions of the 8-orientifold. Type I D-strings wrapping around the circle are T-dualized to the D-particles we wish to consider. By symmetry, these D-particles are originally on the 8-orientifold but we will also consider their dynamics in the perpendicular direction  $X^9$  by allowing for Wilson lines.

In addition to D-particles and orientifolds, the familiar consistency conditions on the RR charges require the presence of 16 D8-branes, the T-dual of the type I D9-branes. We consider only the simple situation of 8 such D8-branes sitting near each of the two 8-orientifolds, so that there is no net RR background charge in the bulk. We can introduce Wilson lines for these D8-branes as well, corresponding to small displacements away from the 8-orientifold. This requires the addition of fermionic bilinears (“bare masses”) to the action for the D-particle coming from the quantization of open strings in the 0-8 sector. This is important for a full cancellation of the vacuum energy of the fast modes.

Consider having  $N$  D-particles. The Chan-Paton indices  $I, J$  will run from 1 to  $2N$  because of the presence of the mirror images. In the 0-0 open

string massless sector, the action of  $\Omega R$  is given by [42]:

$$\begin{aligned} b_{-1/2}^{0,9}|IJ> &\rightarrow -M_{II'}b_{-1/2}^{0,9}|J'I'> M_{J'J}^{-1} \\ \Omega R : \quad b_{-1/2}^{1,2\cdots 8}|IJ> &\rightarrow M_{II'}b_{-1/2}^{1,2\cdots 8}|J'I'> M_{J'J}^{-1} \\ |\alpha IJ> &\rightarrow \gamma_{\alpha\beta}^9 M_{II'}|\beta J'I'> M_{J'J}^{-1}. \end{aligned} \quad (5)$$

A few words of explanation are in order.

The bosonic coordinates are obtained by acting with the operators  $b_{-1/2}^\mu$  on the NS vacuum  $|IJ>$  and the fermionic coordinates are given by the GSO projected R vacuum  $|\alpha IJ>$ , where  $\alpha = (a, \dot{a}) \in \mathbf{8}_s + \mathbf{8}_c$ . The matrix  $M$  is a  $2N \times 2N$  symmetric and unitary matrix whose only non zero entries are those connecting a D-particle with its mirror and the  $16 \times 16$  matrix  $\gamma_9$  is related to the  $32 \times 32$  Dirac matrices of  $\text{spin}(9, 1)$  as  $-\Gamma^0 \Gamma^9 = \gamma_9 \otimes \sigma_3$  with the conventions of [30]. Later we will also need the explicit expression of the matrices  $\gamma_1, \dots, \gamma_9$  in terms of their own  $8 \times 8$  “Pauli” matrices:

$$\gamma_i = \begin{pmatrix} 0 & \sigma_{a\dot{a}}^i \\ \sigma_{bb}^i & 0 \end{pmatrix} \quad i = 1, \dots, 8 \quad \gamma_9 = \begin{pmatrix} \delta_{ab} & 0 \\ 0 & -\delta_{\dot{a}\dot{b}} \end{pmatrix}. \quad (6)$$

Armed with those tools we can easily work out the relevant degrees of freedom by requiring that we keep only states invariant under  $\Omega R$ . The surviving bosonic coordinates are

$$A_{0,9}^{IJ} \quad X_{1,\dots,8}^{IJ} \quad \text{and} \quad x_{1,\dots,8}, \quad (7)$$

where  $A_0$  and  $A_9$  are antisymmetric in the indices  $I$  and  $J$ , giving the gauge group  $SO(2N)$ , whereas the remaining coordinates split into a traceless symmetric representation  $X_i^{IJ}$  and a singlet  $x_i$ . Similarly, the surviving fermionic coordinates are

$$S_a^{IJ} \quad S_{\dot{a}}^{IJ} \quad \text{and} \quad s_{\dot{a}}, \quad (8)$$

where  $S_a^{IJ}$  is in the adjoint (i.e., antisymmetric), and  $S_{\dot{a}}^{IJ}$  is in the traceless symmetric and  $s_{\dot{a}}$  is again a singlet. The bosonic and fermionic singlets describe the collective motion of the centre of mass parallel to the 8-orientifold and we will ignore them in what follows. We can represent the action of  $SO(2N)$  on the various fields by simple commutators or anticommutators; e.g.  $[A_9, X_i]$  is itself traceless and symmetric, and so is  $\{S_a, S_{\dot{a}}\}$ .

The action, the hamiltonian and the 8 supercharges are fixed uniquely by group theory and supersymmetry. It is straightforward to obtain them by

generalizing the formulas of [30, 31, 35]. The action is:

$$\begin{aligned}
S = \int dt \operatorname{Tr} \Bigg\{ & \frac{1}{\lambda_{I'}} \left( \frac{1}{2} \dot{X}_i^2 - \frac{1}{2} \dot{A}_9^2 - \dot{A}_9 [A_0, A_9] + \dot{X}_i [A_0, X_i] - \frac{1}{2} [A_0, A_9]^2 \right. \\
& + \frac{1}{2} [A_0, X_i]^2 - \frac{1}{2} [A_9, X_i]^2 + \frac{1}{4} [X_i, X_j]^2 \Bigg) + \frac{i}{2} \left( S_{\dot{a}} \dot{S}_{\dot{a}} - S_a \dot{S}_a - S_a [A_0, S_a] \right. \\
& \left. \left. + S_{\dot{a}} [A_0, S_{\dot{a}}] + S_a [A_9, S_a] + S_{\dot{a}} [A_9, S_{\dot{a}}] - 2X_i \sigma_{a\dot{a}}^i \{S_a, S_{\dot{a}}\} \right) \right\}. \quad (9)
\end{aligned}$$

From (9), the hamiltonian and the Gauss' law can be derived. We present only the hamiltonian in the temporal gauge  $A_0 = 0$ . Denoting by  $E_9$  and  $P_i$  the momenta canonically conjugated to  $A_9$  and  $X_i$  we have:

$$\begin{aligned}
H = \operatorname{Tr} \Bigg\{ & \lambda_{I'} \left( \frac{1}{2} \dot{P}_i^2 - \frac{1}{2} \dot{E}_9^2 \right) + \frac{1}{\lambda_{I'}} \left( \frac{1}{2} [A_9, X_i]^2 - \frac{1}{4} [X_i, X_j]^2 \right) \\
& \left. + \frac{i}{2} \left( -S_a [A_9, S_a] - S_{\dot{a}} [A_9, S_{\dot{a}}] + 2X_i \sigma_{a\dot{a}}^i \{S_a, S_{\dot{a}}\} \right) \right\}. \quad (10)
\end{aligned}$$

Finally we can write down the 8 (and not 16 as in the type IIA case) supercharges whose anticommutation relations give (10) up to a gauge transformation:

$$Q_a = \operatorname{Tr} \left\{ \sqrt{\lambda_{I'}} \left( \sigma_{a\dot{a}}^i S_{\dot{a}} P_i - S_a E_9 \right) + \frac{1}{2\sqrt{\lambda_{I'}}} \left( \sigma_{ab}^{ij} X_i [S_b, X_j] + \sigma_{a\dot{a}}^i S_{\dot{a}} [A_9, X_i] \right) \right\}. \quad (11)$$

Equations (9,10,11) define the system we wish to study in the following sections, for general  $N$  and for  $N = 2$  (gauge group  $SO(4)$ ) in particular.

## 4 Bound states and flat directions

The moduli space of the system is defined by the gauge inequivalent solutions to

$$[A_9, X_i] = [X_i, X_j] = 0. \quad (12)$$

The flat directions associated with  $A_9$  play a special role in that they define the position of the D-particles relative to the 8-orientifold or, equivalently, relative to their mirror images. To fix the ideas, consider the case of two

D-particles. The two elements of the Cartan subalgebra of  $SO(4)$  can be written as

$$H^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad H^2 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \quad (13)$$

The configuration  $A_9 = a_9^1 H^1 + a_9^2 H^2$  represents two D-particles whose distances from the 8-orientifold are  $a_9^1$  and  $a_9^2$  respectively. In the generic situation, the only traceless symmetric matrix that commutes with  $A_9$  above is

$$M^0 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (14)$$

and the coordinates  $x_i$  in  $X_i = x_i M^0$  represent the relative distance of the D-particles in the eight directions parallel to the 8-orientifold. These coordinates, together with those of the centre of mass, fix the configuration of the system.

In general for  $SO(2N)$  there will be  $N - 1$  neutral directions (corresponding to a weight  $(0, \dots, 0)$  in Dynkin basis) that give rise to flat directions. This is precisely what is needed to account for the relative positions of the  $N$  D-particles parallel to the 8-orientifold. It is a familiar fact that, for particular “degenerate” points in the Cartan subalgebra, more general configurations for the  $X_i$ ’s are possible. That is, new flat directions open up that reduce the rank of the gauge group. In terms of weights we must then turn on the  $X_i$  in directions that are not neutral, see e.g [53]. Semiclassically, this corresponds to “locking” the system in a specific configuration along the ninth coordinate, but it is not clear whether it is possible to prepare the system in such configurations. An interesting example of this phenomenon is the configuration  $X_i = x_i \tilde{M}$  that for

$$\tilde{M} = \frac{1}{\sqrt{2(a^2 + b^2 + c^2 + d^2)}} \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \quad a + b + c + d = 0 \quad \text{all different} \quad (15)$$

breaks the gauge group entirely, therefore locking the two D-particles on the 8-orientifold.



The existence of resonances above BPS saturation can be investigated with exactly the same tools as in [30] to which we refer for all the details. What one does is to fix the temporal gauge  $A_0 = 0$  and to separate the  $18N^2 + 7N - 8$  bosonic directions<sup>8</sup> into  $2N^2 + 7N - 8$  slow modes<sup>9</sup> and the remaining  $16N^2$  fast modes, i.e., those modes for which the hessian of the bosonic potential is (generically) non degenerate.

The ground state energy for the bosonic fast modes is cancelled by the ground state energy of the fermions, as expected by supersymmetry, leaving only a confining linear potential independent of the string coupling. This is interpreted as coming from on shell strings stretching between the two D-particles. To be precise, this cancellation is not exact unless we also add the contribution from the D8-branes as we will show in the next section.

When the linear potential is included in the effective action for the slow modes, as prescribed by the Born-Oppenheimer approximation, one sees the existence of bound states above threshold whose dependence on the string coupling can be computed exactly by the same scaling argument as in [30]. The following table summarizes the scaling of the mass  $M$ , the binding energy  $E$ , the orbital velocity  $v$  and the orbital momentum  $p$  in the units appropriate to the relevant string theories and “M-theory” in the notation of Horava and Witten [11]; notice the simple dependence of all the quantities in the M-theory units.

For later use, we also include the heterotic string units in the table. We will come back to this in section 6.

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<sup>8</sup> $\dim(\text{Adj}(SO(2N))) + 8 \dim(\text{Sym}(SO(2N))) = 18N^2 + 7N - 8$ .

<sup>9</sup>The  $N + 8(N - 1)$  dimensional moduli space plus  $\dim(SO(2N)) - \text{rank}((SO(2N))$  gauge directions.

	$I'$	$E_8 \times E_8$	M-theory
$M$	$1/\lambda_{I'}$	$1/R_{E_8}$	$1/R_{11}$
$E$	$\lambda_{I'}^{1/3}$	$R_{E_8}/\lambda_{E_8}^{2/3}$	$R_{11}$
$v$	$\lambda_{I'}^{2/3}$	$R_{E_8}/\lambda_{E_8}^{1/3}$	$R_{11}$
$p$	$1/\lambda_{I'}^{1/3}$	$1/\lambda_{E_8}^{1/3}$	1

## 5 Scattering and the metric on the moduli space

We now move on to the related issue of the scattering of these objects. We use the same techniques as in [31, 35] i.e., the computation of the potential and the quantum mechanical phase shifts in the eikonal approximation. There are two simple situations we wish to consider: the first is the scattering of one D-particle off “the plane”<sup>10</sup>, the second is the scattering of two D-particles against each other and parallel to the plane. In the first case, we will find a potential  $\mathcal{V} \approx v^2 r^{-3}$ , where  $v$  is the velocity of the D-particle towards the plane and  $r$  the distance between the two objects. The fact that the potential vanishes at zero velocity is due to the 8 supersymmetries of the system and to the cancellation between the contributions from the 8-orientifold and the D8-branes. In the second case, we find that the potential is of order  $v^4 r^{-7}$  exactly as in the type IIA case.

In the case of a D-particle moving towards the plane, the relevant gauge group is  $SO(2)$ , whose unique generator we represent by  $(i/2)\sigma^2$ . The singlets described in section 3 decouple as usual but we are still left with two more

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<sup>10</sup>We refer to the composite system of 8-orientifold plus 8 D8-branes and their mirror images simply as “the plane” for conciseness.

bosonic degrees of freedom for each parallel direction:  $X_i = 1/2(x_i\sigma^1 + \tilde{x}_i\sigma^3)$ . The fermions are subjected to the same treatment as the bosons, with eight real Grassmann degrees of freedom from  $S_a$  and sixteen from  $S_{\tilde{a}}$ . The geometry of this configuration is “degenerate” in the sense that the impact parameter is necessarily zero as we consider the scattering off an eight dimensional object in nine dimensional space. The collision is therefore “head on” and one might be skeptical about the validity of the eikonal approximation. We shall argue however that, due to the cancellations involved, it is possible to extrapolate the technique to this situation.

The background gauge  $\bar{A}_9 = (i/2)vt\sigma^2$  represents a D-particle moving towards the plane. This yields sixteen massive bosonic degrees of freedom ( $x_i$  and  $\tilde{x}_i$ ) and sixteen fermionic ones ( $S_{\tilde{a}}$ ). There are no massive modes coming from gauge fields, ghosts or  $S_a$  because the gauge group is abelian. The contribution to the one loop integral<sup>11</sup> coming from the 8-orientifold alone is thus ( $\tau = it$  and  $\gamma = -iv$ ):

$$\mathcal{A}_{\text{orientifold}} = \det(-\partial_\tau^2 + \gamma^2\tau^2)^{-8} \det \begin{pmatrix} \partial_\tau & \gamma\tau \\ \gamma\tau & \partial_\tau \end{pmatrix}^4. \quad (16)$$

Expression (16) is badly divergent, but we can improve the situation by considering the effective potential  $\mathcal{V}_{\text{orientifold}}(v, r)$  defined through

$$\mathcal{A}_{\text{orientifold}} = \int_{-\infty}^{+\infty} dt \mathcal{V}_{\text{orientifold}}(v, vt), \quad (17)$$

that is, using analytic continuation for the integrals,

$$\mathcal{V}_{\text{orientifold}}(v, r) = v \int_0^{+\infty} \frac{ds}{\sqrt{\pi s}} e^{-sr^2} \frac{4 - 2\cos vs}{\sin vs} \approx r \left( -4 + \frac{v^2}{2r^4} + O\left(\frac{v^4}{r^8}\right) \right). \quad (18)$$

However, this is not the end of the story. One must also take into account the 16 fermionic modes from the D8-branes and their mirrors. This gives a contribution

$$\mathcal{A}_{\text{D8-brane}} = \det \begin{pmatrix} \partial_\tau & \gamma\tau \\ \gamma\tau & \partial_\tau \end{pmatrix}^4, \quad (19)$$

that always goes together with (16), yielding an extra term in the effective

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<sup>11</sup>The “would be” phase shift if it were possible to set  $b \neq 0$ .

potential:

$$\mathcal{V}_{\text{D8-brane}}(v, r) = -v \int_0^{+\infty} \frac{ds}{\sqrt{\pi s}} e^{-sr^2} \frac{2 \cos vs}{\sin vs} \approx r \left( 4 + \frac{v^2}{2r^4} + O\left(\frac{v^4}{r^8}\right) \right). \quad (20)$$

Adding the two contributions together the velocity independent piece linear in  $r$  cancels and the total effective potential becomes, to leading order,

$$\mathcal{V}(v, r) = \mathcal{V}_{\text{orientifold}}(v, r) + \mathcal{V}_{\text{D8-brane}}(v, r) \approx \frac{v^2}{r^3} + \dots \quad (21)$$

What we see here is a reflection in the low energy theory of the necessity of having eight D8-branes together with the 8-orientifold. Note also that if we allow the D8-branes to be positioned off the 8-orientifold, the linear potential will still cancel since the center of mass for the D8-branes and their mirror images is always right on the 8-orientifold. That the cancellation persists even when the D8-branes are far away from the 8-orientifold/D-particle system is of course due to the potential being precisely linear. It is important that the linear potential cancels as we move away from the system. We would otherwise risk a bad IR-divergence.

Another example where care is needed is the case of a D-particle in the vicinity of a single D8-brane. Naively there would be a linear force on the D-particle when we integrate out the fermionic open strings. However, this system would suffer from an IR-divergence. A way to fix this is to put another D8-brane at infinity. Its only effect would be to cancel the linear force and we thereby reproduce the result of [50].

It is important to observe that the result for the potential above, i.e.  $v^2/r^3$ , is only true for small  $r$ . At long distance it is not protected against corrections from excited open strings, and we expect that it will smoothly go over into  $v^2/r^7$ . Note that this is really an interaction between a D-particle and its mirror image. Following [35] one can deduce that for a 0-0 system  $v^4/r^7$  is protected, for a 0-4,  $v^2/r^3$  is protected, while for 0-8,  $r$  is protected.

Moving on to a less pathological situation, let us consider the scattering of two D-particles with velocity  $v$  and impact parameter  $b$  in the 8 dimensional space parallel to the plane. Let us therefore consider the background gauge  $\bar{X}_1 = vtM^0$  and  $\bar{X}_2 = bM^0$  in the  $SO(4)$  system of section 4 ( $M^0$  given by (14)). From (9) we can read off the massive modes and their contribution to

the phase shift <sup>12</sup>:

$$e^{i\delta} = \det(-\partial_\tau^2 + \gamma^2 \tau^2 + b^2)^{-12} \det(-\partial_\tau^2 + \gamma^2 \tau^2 + b^2 + 2\gamma)^{-2} \\ \times \det(-\partial_\tau^2 + \gamma^2 \tau^2 + b^2 - 2\gamma)^{-2} \det \begin{pmatrix} \partial_\tau & \gamma\tau - ib \\ \gamma\tau + ib & \partial_\tau \end{pmatrix}^{16}. \quad (22)$$

But  $\delta$  from (22) is just twice the phase shift computed in [35]! We can therefore carry through all their results, in particular the flatness of the metric on the moduli space. Actually, as we move away from the 8-orientifold, half of these modes become heavy and decouple, yielding the type IIA result of [35].

## 6 The heterotic string

In the introduction we promised that our results would also tell us something about the heterotic string. Let us begin by a discussion of what we might expect to find.

The system we are considering is, from M-theory point of view, a compactification from eleven to nine dimensions. Note that nine is the minimal dimension where we can find all string theories through M-theory compactification.

There are two main types of such compactifications. The simplest is to put M-theory on a torus,  $S^1 \times S^1$ , or equivalently type IIA on a circle. T-duality then gives you the IIB string. The D-particles that were studied in [30, 31, 35] clearly survives this compactification and can be further studied in this nine dimensional setting. The other compactification involves a  $Z_2$  projection on one of the  $S^1$  in the torus. A type IIB string becomes a type I and a type IIA a type I'. The latter is the system that we have been studying in this paper. Exchanging the  $S^1/Z_2$  and the  $S^1$  produces the heterotic strings.

A membrane in M-theory can give rise to several different states in the nine dimensional string theory with masses given by, in the notation of [8],

$$M^2 = \frac{l^2}{R_{10}^2} + \frac{m^2}{R_{11}^2} + n^2 R_{10}^2 R_{11}^2 \quad (23)$$

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<sup>12</sup>Actually, the most practical way to determine the masses of the massive modes, is to make a group theoretical argument, like the one in [53], based on the weights of the representations.

in “M-theory” units. This is true both for the type II side and the heterotic/type I side. In the latter case we must make a  $Z_2$  projection on the tenth direction. Depending on the specific string theory the states have very different interpretations. In I’ units the masses are

$$M_{I'}^2 = \frac{l^2}{R_{I'}^2} + \frac{m^2}{\lambda_{I'}^2} + n^2 R_{I'}^2, \quad (24)$$

while in heterotic  $E_8 \times E_8$  we have

$$M_{E_8}^2 = \frac{l^2}{\lambda_{E_8}^2} + \frac{m^2}{R_{E_8}^2} + n^2 R_{E_8}^2. \quad (25)$$

In type I’ we have two 8-orientifolds separated by  $R_{I'}$ , one for each fixed point of  $Z_2$ . In addition there are 16 D8-branes whose positions determine the space time gauge group. The coupling constant  $\lambda_{I'}$  is determined by the remaining compact  $S^1$ . In the case of the heterotic string, the coupling constant  $\lambda_{E_8}$  is instead determined by the distance between the two 8-orientifolds. Our D-particle, which has a non zero  $m$  quantum number, is hence a nonperturbative state in the I’ theory but just a Kaluza-Klein mode of the heterotic string in the heterotic theory.

The D-particles will interact with each other, with their mirror images, and also with the the D8-branes and their mirror images. States with  $l$  quantum numbers are not stable in the heterotic/I case. This is due to momentum not being conserved in the tenth direction; here we have an interval not a circle. This is reflected in e.g. a velocity dependent force transverse to the 8-orientifolds. This we indeed verified in the previous section.

The calculations that we performed in the previous section for scattering parallel to the plane is effectively nine dimensional, and if we can extend them to include two planes we should be able to interpret the results in terms of a nine dimensional heterotic string theory. What do we know about the dynamics of black holes in such a theory?

Let us consider the (gravitational) bosonic part of the effective action for a heterotic string in nine dimensions. This is obtained through dimensional reduction and is given by

$$S = \frac{1}{2} \int d^9x \sqrt{-g} \left( R - \frac{1}{2}(d\xi)^2 - \frac{1}{2}(d\phi)^2 - \frac{1}{4}e^{-\frac{4}{\sqrt{7}}\xi}(d\mathcal{A})^2 - \frac{1}{4}e^{-\phi+\frac{3}{\sqrt{7}}\xi}(dA)^2 \right) \quad (26)$$

where  $\xi \sim g_{99}$ ,  $\mathcal{A}_\mu \sim g_{\mu 9}$  and  $A_\mu \sim B_{\mu 9}$  and we have set the two form  $B_{\mu\nu} = 0$  since we are interested in  $p = 0$  solitons. One-scalar-solutions can be obtained in two different ways, either by taking a linear combination of  $\phi$  and  $\xi$  or by just setting  $\phi = A = 0$ . The first of these has  $\Delta = 2$ , the second  $\Delta = 4$ , where  $\Delta$  is an invariant given by [12, 54, 55]

$$\Delta = a^2 + \frac{2(p+1)(D-p-3)}{D-2} = a^2 + \frac{12}{7}, \quad (27)$$

for  $p = 0$  and  $D = 9$ . The quantity  $a^2$  is obtained from  $e^{-\mathbf{a} \cdot \mathbf{\Phi}}$ , where  $\mathbf{\Phi} = (\phi, \xi)$ . The linear combination  $\mathbf{a} \cdot \mathbf{\Phi}$  is the scalar field that we allow to be nonzero. One also has that  $\Delta = 4/N$  where  $N$  is the number of participating field strengths. The black hole with  $\Delta = 4$  is the one which is described by a D-particle because it's a purely Kaluza-Klein mode.

What will the force between two such black holes be? We will follow [12] and find that the force starts only at  $v^4$ . This is a general result for  $\Delta = 4$  as can be seen easily from the following calculation. The metric of an extremal black hole in  $D$  dimensions is, (see e.g. [56, 57]),

$$ds^2 = f^{\frac{N}{D-2}} (-f^{-N} dt^2 + dr^2 + r^2 d\Omega_{D-2}^2), \quad (28)$$

where

$$f = 1 + \frac{r_e^{D-3}}{r^{D-3}}. \quad (29)$$

The dilaton is given by

$$e^{2\phi} = f^{-aN}. \quad (30)$$

The action for a black hole moving in the background of another, identical black hole, is

$$S = -m \int dt \left( \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu e^{a\phi}} - \dot{x}^\mu A_\mu \right). \quad (31)$$

From (31) we can read off the force. By supersymmetry the gauge field  $A_\mu$  is such that the velocity independent force cancels. Let us concentrate on velocity dependent forces. We find:

$$\sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu e^{a\phi}} \sim g_{tt}^{1/2} e^{a\phi/2} - \frac{1}{2} e^{a\phi/2} \frac{g_{rr}}{g_{tt}^{1/2}} v^2 + O(v^4). \quad (32)$$

The coefficient of  $v^2$  is hence given by

$$-\frac{1}{2} f^{\frac{N}{4}(4-\Delta)}, \quad (33)$$

and thereby we find that the force vanishes for  $\Delta = 4$  but is nonzero for  $\Delta \neq 4$ .

This is consistent with what we calculated in the previous section for scattering parallel to the 8-orientifold. In the heterotic limit there is an extra subtlety due to the fact that we have two 8-orientifolds that come close. This makes it possible to have extra open strings stretching between the D-particles. We must in fact sum over strings that bounce between the two 8-orientifolds an arbitrary number of times. With one 8-orientifold we only needed to consider a string stretching directly between the two D-particles and one that bounced off the 8-orientifold once (equivalently connected to the mirror image of the other D-particle). This sum converts the  $1/r^7$  to  $1/r^6$  at weak coupling.

Let us recall the table in the previous section! The fact that the length of the interval  $R_{10}$  does not appear in the last column, makes it more plausible to extrapolate the results to the heterotic case. The fact that the orbital momentum is of order one in M-theory units is the direct analogue of the fact noted in [31, 35] that the D-particles allow one to probe distances of the order of the eleven dimensional Planck length, which is smaller than the string length in *both* the type I' and the heterotic limit:  $l_{\min} = \lambda_{I'}^{1/3} \ll 1$  in the limit where the type I' description applies and  $l_{\min} = \lambda_{E_8}^{1/3} \ll 1$  in the limit where the heterotic string is weakly coupled.

One should note that the calculation is valid only for  $v$  small (we are considering Yang-Mills, not Born-Infeld), which for the heterotic string means that  $R_{E_8}/\lambda_{E_8}^{1/3}$  is small. Luckily, in nine dimensions, there exists a region where both the type I' and the heterotic string are weakly coupled. But since  $R_{E_8} = \lambda_{I'}^{2/3} l_{11}$  and  $R_{I'} = \lambda_{E_8}^{2/3} l_{11}$  it follows that the compact dimension must be smaller than the eleven dimensional Planck length for a weakly coupled type I', and the same must be true for the distance between the orientifolds to also have a weakly coupled heterotic string. While our quantum mechanical system still should provide a good description, the Born-Oppenheimer approximation would break down for strings stretching from a D-particle to its own mirror. However, these strings were only relevant for transverse scattering and not for the parallel scattering that we are considering here, so that there is some hope that some of these results carry over to the heterotic string.

The black hole with  $\Delta = 2$  is perhaps even more interesting since it has a non flat moduli space. This black hole must correspond to a heterotic



string that has both winding and momentum in the compact direction since it couples to both  $A$  and  $\mathcal{A}$ . Such an example was constructed in [58, 59] where, indeed, a non flat moduli space was verified. If we consider the mass formulae above we see that such a system must correspond to a membrane wrapped around  $R_{11}$  and stretched between the 8-orientifolds. Can such an object be constructed using D-particles?

In [30] a bound state of two D-particles was studied and it was found that the bound state had a total mass given by

$$\frac{2}{R_{11}} + \epsilon R_{11} \quad (34)$$

in M-theory units. The second term is due to the presence of a string stretching between the two D-particles. Here we would like, in analogy with (23) and (24), to interpret the string as a membrane wound around the compact dimension.  $\epsilon$  above is the expectation value of the distance between the D-particles and  $\epsilon R_{11}$  is hence the area of the membrane. We conclude, then, that a string stretching between two D-particles is actually a membrane wrapped around  $R_{11}$ . Note that this state is a non-BPS state, with no topological protection and therefore not stable. It is interesting to note that what is essentially only a scaling argument, provides evidence that the D-particle system not only knows about M-theory through the eleven dimensional Planck length, but also about membranes.

With these observations in mind, it is reasonable to expect that it should be possible to construct stable states with both winding and momentum corresponding to the  $\Delta = 2$  black holes using D-technology. We should then have strings stretching between the 8-orientifolds, threaded by D-particle beads.

## 7 Conclusions

In this paper we have investigated the quantum mechanical system that describes D-particles in the presence of 8-orientifolds and D8-branes. For consistency each 8-orientifold must be matched by 8 D8-branes to cancel any linear potential. This we saw reflected in the quantum mechanics of the D-particles.

We have calculated the scattering of D-particles moving parallel to the 8-orientifold/D8-brane system finding  $v^4/r^7$  forces only. In addition we con-

sidered D-particles moving in the transverse direction. In that case the force turned out to be proportional to  $v^2/r^3$ .

While this is obviously an example of type I' dynamics, we have argued that the result also can be interpreted as a calculation of the scattering of  $\Delta = 4$ , heterotic Kaluza-Klein black holes.

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